Статистическая обработка сигналов. Введение

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Курс “Статистическая обработка временных рядов”

Сентябрь 2018

What is a Signal?

Definition

Signal is a real or complex valued function of one or more real variables

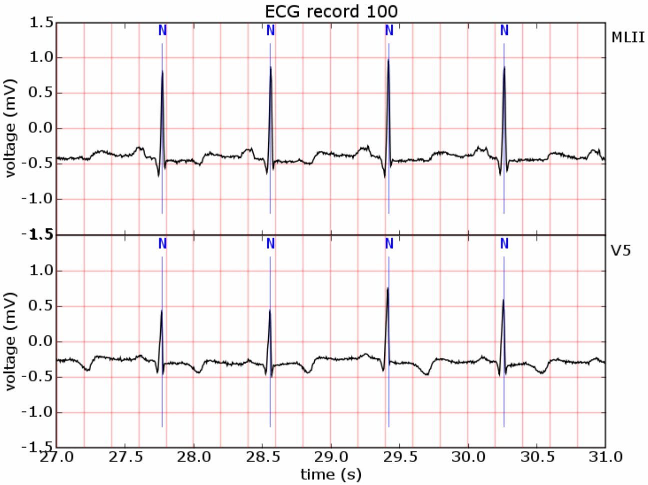
When the function depends on a single variable, the signal is said to be one-dimensional. Otherwise, the signal is multidimensional

Examples of independent variables:
Time, distance, position, temperature, pressure, etc.

Examples of signals:
1-D signals: speech, music, chirping of birds, the electromagnetic field emanating from a transmitting antenna, variation of light intensity in an optical fiber, etc.
2-D signals: image
3-D signals: video stream

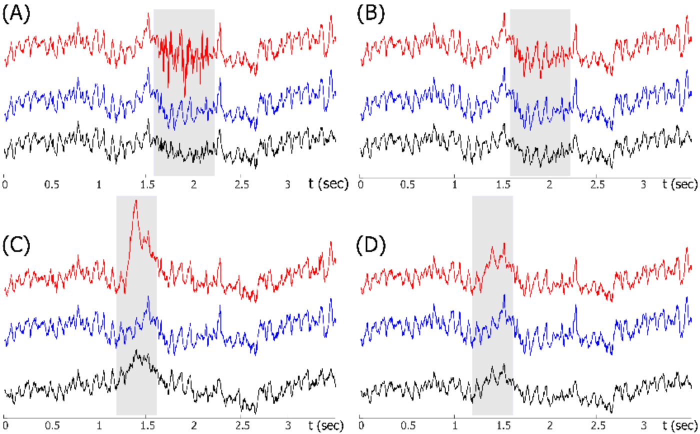
Example: ECG

Biological signals: electrocardiography (ECG)



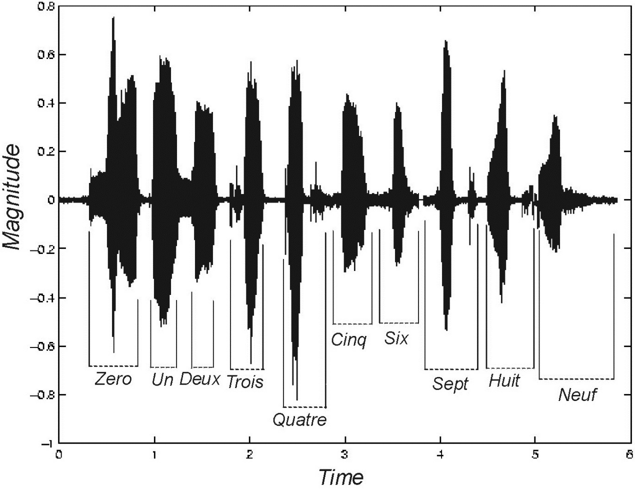
Example: EEG

Biological signals: electroencephalography (EEG)



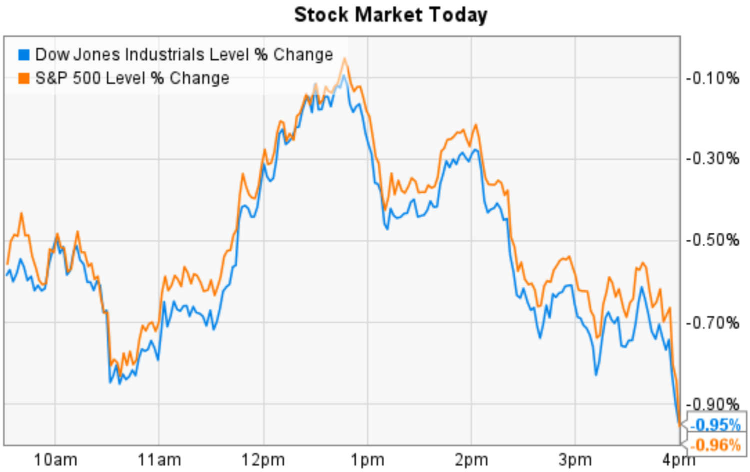
Example: Speech

Speech signal



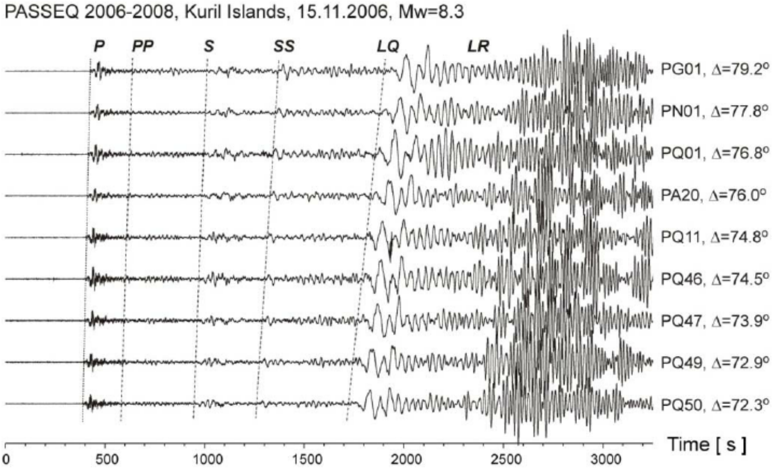
Example: Stock Data

Financial signal: stock data



Example: Seismogram

Seismic signal: seismogram



Types of Signals

- One-dimensional and multidimensional
- Deterministic and random
- Continuous and discrete
- Analog and digital
- Periodic and aperiodic
- Stationary and non-stationary
- Finite and infinite
- Real-valued and complex-valued
- Monofrequency and multifrequency
- ...

Deterministic and Random Signals

A signal is deterministic if it is exactly predictable for the time span of interest. Deterministic signals can be described by mathematical models

Examples of deterministic signals:

$x(t) = A\cos(\omega t)$, where $A = const$

$x(t) = \exp(-kt)\cos(\omega t + \varphi_0)$

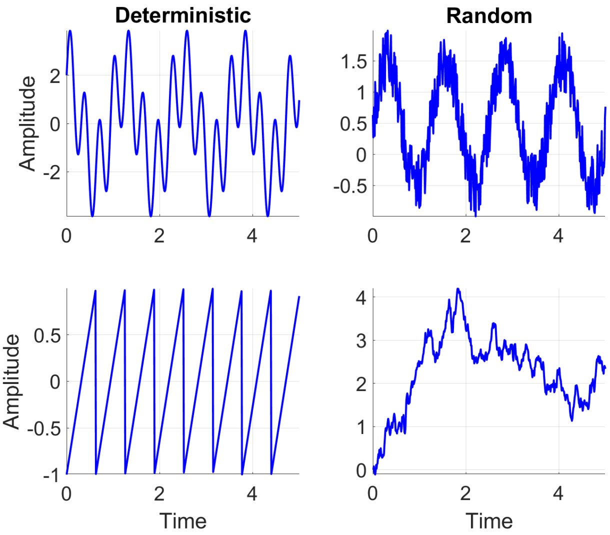
A signal is random (stochastic) if it takes on values by chance according to some probabilistic model. Stochastic signals can be described by statistical properties and probabilities

Examples of stochastic signals:

$X(t) = A\cos(\omega t)$, where A is a random variable, $A \sim N(0,1)$

$$X(t) = \begin{cases} X(t-1)+1, & \text{with probability } p, \\ X(t-1)-1, & \text{with probability } q, \\ X(t-1), & \text{with probability } 1-p-q \end{cases}$$

Deterministic and Random Signals. Illustration



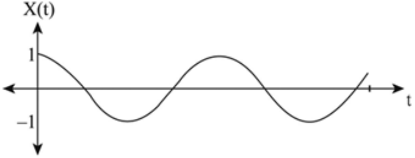
Continuous and Discrete Signals

Let's denote $x(t)$ is a signal, t is independent variable

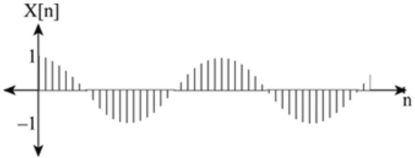
A signal $x(t)$ is continuous if the independent variable t takes values in an interval (e.g., $t \in (-\infty, \infty)$, $t \in [0, \infty)$, $t \in [T_1, T_2]$)

A signal $x(t)$ is discrete if t takes values in a countable set (e.g., $t \in \{0, T, 2T, 3T..., \}$, $t \in \{..., -1, 0, 1, ...\}$, $t \in \{1/2, 3/2, 5/2, ...\}$)

Continuous



Discrete



Analog and Digital Signals

The value of a signal $x(t)$ at a specific value of the independent variable (time) t is called its amplitude at time t

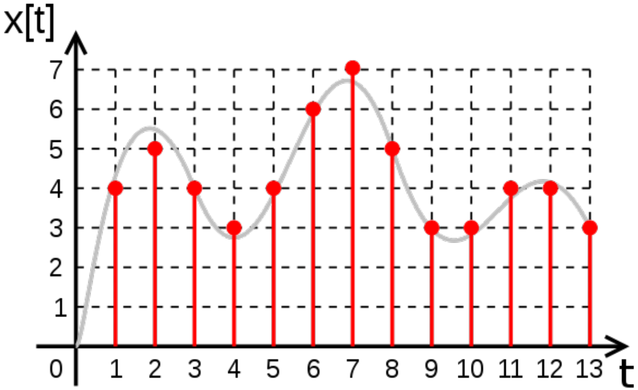
Analog signal is a continuous-time signal with a continuous amplitude

Digital signal is a discrete time signal with discrete valued amplitudes

Analog signals are typically represented by physical quantities as a voltage, electric current, or electric charge around components in the electronic devices

Digital signals can be generated by quantizing the amplitude of analog signal at specific points in time or naturally (e.g., stock prices)

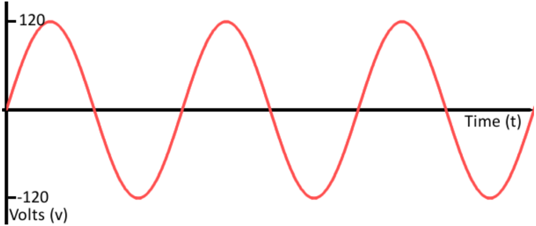
Analog and Digital Signals. Illustration 1



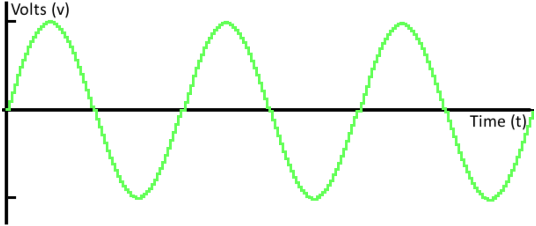
A digital signal is an abstraction that is discrete in time and amplitude, it only exists at certain time instants

Analog and Digital Signals. Illustration 2

Analog waveform



Digital waveform



Analog waves are smooth and continuous, digital waves are stepping, square and discrete

Sampling

Definition

Sampling is the process of recording an analog signal at regular discrete moments of time

The time interval T between samples is called the sampling interval (sampling period)

Let's $x(t)$ to be an analog signal. The corresponding sampled signal:

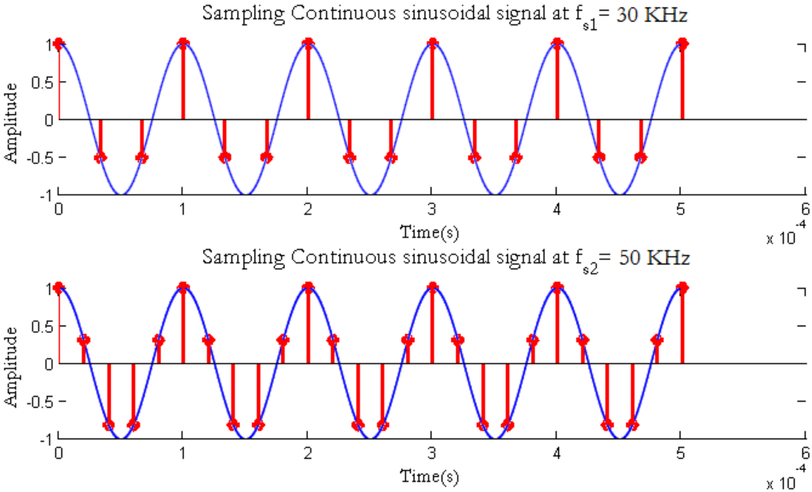
$$x(T), x(2T), x(3T), ...$$

The sampling frequency (sampling rate) f_s is the average number of samples obtained in one second (samples per second):

$$f_s = \frac{1}{T}$$

A subsystem or operation that extracts samples from a continuous signal is called the sampler

Sampling. Illustration



Quantization

Definition

Quantization is the process of mapping signal amplitudes to output values in a finite discrete set

A device or algorithmic function that performs quantization is called a quantizer

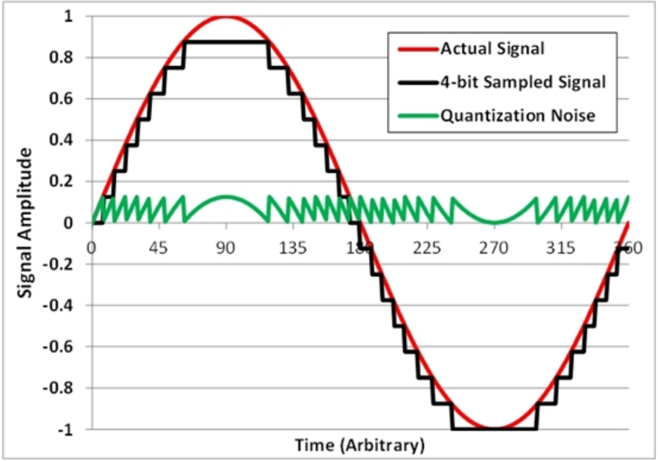
The difference between an input amplitude and its quantized value is referred to as quantization error

Quantizing a sequence of amplitudes produces a sequence of quantization errors which is sometimes modeled as an additive random signal called quantization noise because of its stochastic behavior

The more levels a quantizer uses, the lower is its quantization noise

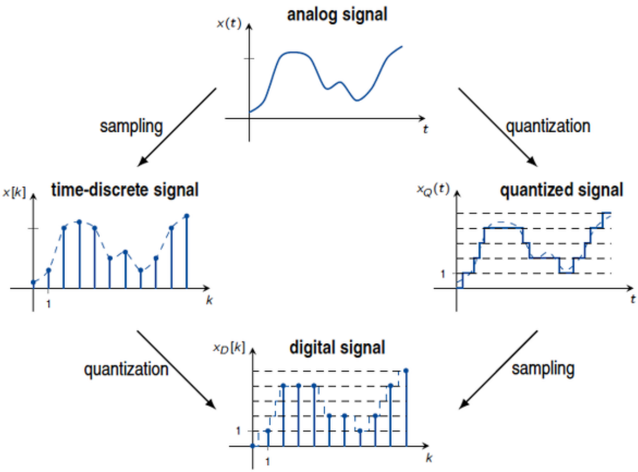
Quantization. Illustration

The simplest way to quantize a signal is to choose the digital amplitude value closest to the original analog amplitude



Digitization

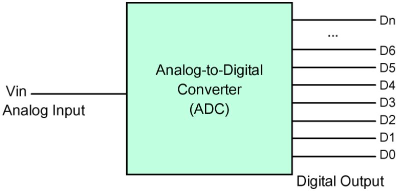
Digitization means sampling and quantization

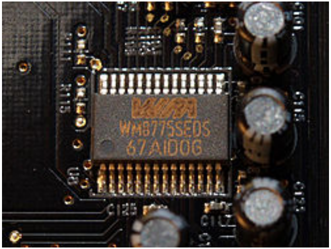


Analog-to-Digital Converter (ADC)

ADC converts a continuous-time and continuous-amplitude analog signal to a discrete-time and discrete-amplitude digital signal

Applications: in music recording devices, in digital imaging systems, in radar systems to convert signal strength to digital values for subsequent signal processing, in digital storage oscilloscopes, etc.



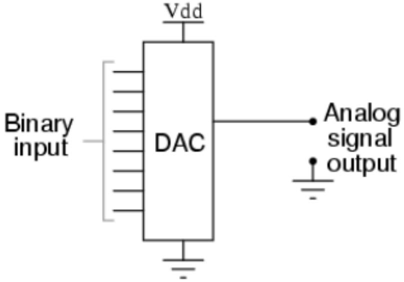


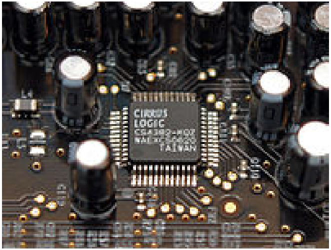
ADC WM8775SEDS Wolfson Microelectronics placed on a X-Fi Fatality Pro sound card

Digital-to-Analog Converter (DAC)

DAC converts digital signal to analog signal

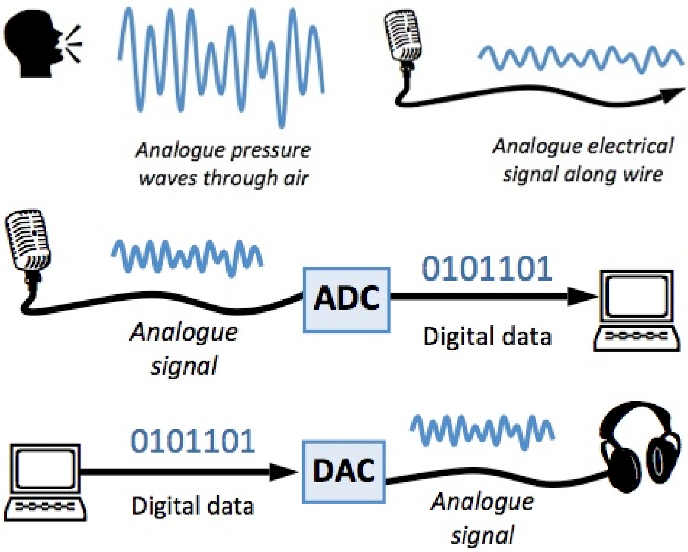
Applications: in music players to convert digital data streams into analog audio signals, in televisions and mobile phones to convert digital video data into analog video signals which connect to the screen drivers to display monochrome or color images, etc.



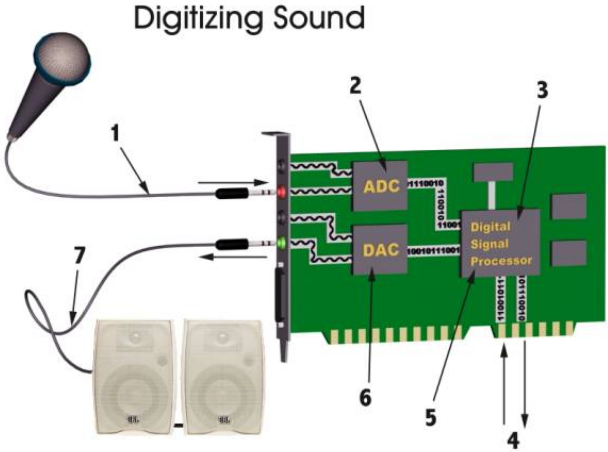


8-channel Cirrus Logic CS4382 DAC in a sound card

ADC and DAC. Illustration



Digitizing Sound



As soon as sound is digitized we can apply algorithms, manipulate, edit, make copies without losing any quality

Signal Processing

Signal processing is the analysis, synthesis and modification of signals

Applications of signal processing:

- Audio signal processing
- Speech signal processing
- Image processing
- Video processing
- Financial signal processing
- Biomedical signal processing
- Radar signal processing
- Seismic data analysis
- Communication systems
- Control systems
- ...

History of Signal Processing

- 1948 Dawn of digital signal processing era
  C.Shannon published “A mathematical theory of communication”; modern digital methods of spectrum estimation were introduced; IEEE Signal Processing Society was established
- 1950s Exciting time to be an audio engineer
  Automobile phonograph systems (Chrysler Corp.); hi-fi movement; stereo records were introduced; first transistor radio (Regency Company)
- 1960s Going digital
  Digital communication systems; IBM 360; numerical simulations of wave filters; discovery of FFT algorithm; first textbook on digital signal processing; digital filters in analysing the reflected radar signals

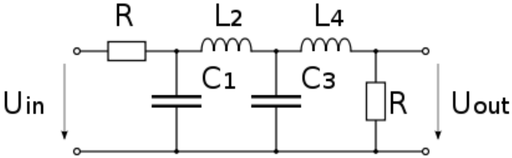
History of Signal Processing

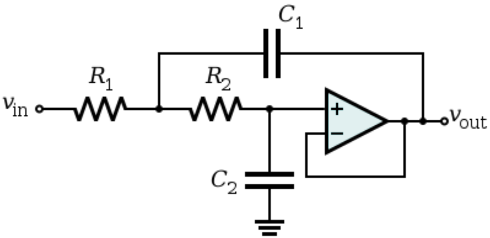
- 1970s Third industrial (digital) revolution
  Digital processing of speech signals; speech synthesized by computer; works on automatic speaker recognition; development of DSP hardware, DSP in space industry (encoding, reconstruction and improvement of space images); DSP of radar and sonar signals
- 1980s Etched in silicon
  Single-chip DSP; CD players; digital fax machines; DSP in medicine (computerized tomography, MRI); space communications (NASA's Deep Space Network); wavelets theory development
- 1990s Breakthrough
  DSP business grows up to $2.2 bln.; digital photography and digital cameras; variety of DSP chips; optical fibers; digital means have displaced analog means for recording, processing, transmission, and reproduction of audio; neural networks for signal processing
- 2000s Signals everywhere
  Smartphones; speech recognition; image processing; autonomous driving; intelligent appliances

Analog Signal Processing

Analog signal processing (ASP) uses analog circuit elements such as resistors, capacitors, transistors, diodes etc.

Examples of analog circuits:





Digital Signal Processing

Digital signal processing (DSP) relies on numerical calculations for digital signals

Scheme of digital signal processing

Digital signal processing can be implemented on digital signal processors or general-purpose computers

Digital signal processor is a specialized microprocessor with its architecture optimized for the operational needs of digital signal processing

Analog vs Digital Signal Processing

Main advantages of digital signal processing:

- Flexibility
  Same hardware can be used to do various kind of signal processing operation, while in analog signal processing one has to design a system for each kind of operation
- Repeatability
  The same signal processing operation can be repeated again and again giving same results, while in analog systems there may be parameter variation due to change in temperature or supply voltage

Main advantages of analog signal processing:

- Doesn't add quantization error and quantization noise
- Doesn't imply sampling problems like aliasing
- Doesn't require neither ADC nor DAC
- Fast and real-time processing

Statistical Signal Processing

Statistical signal processing (SSP) is an approach which treats signals as stochastic processes, utilizing their statistical properties to perform signal processing tasks

In reality most real signals are stochastic processes due to:

- Unknown signal parameters (e.g., delay of radar return)
- Environmental noise (ambient electromagnetic fields, radar jamming)
- Sensor noise (grainy images, old phonograph recordings)
- Variability inherent in nature (stock market)

Typical operations in SSP:

- Spectral estimation
- Modelling of time series
- Prediction
- Filtering